NUMERICAL ANALYSIS: INTEGRATION (PAPER VIII GROUP B)- THIRD YEAR
Lecture 02

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1 Statement of Simpson's one-third formula

Let $y_0, y_1, y_2, ..., y_n$ be $n = 2m+1$ values of the function $y = f(x)$ corresponding to $n(2m+1)$ equidistant arguments $a = x_0, x_1(= x_0+h), x_2(= x_0+2h), ..., x_n = b$, h being the length of equal spacing, then

$$\int_a^b f(x)dx = \int_{x_0}^{x_n} f(x)dx = \frac{1}{3}h[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + ... + y_{n-1}) + 2(y_2 + y_4 + y_6 + ... + y_{n-2})]$$

Error: The error estimate ϵ, involved in Simpson's one -third formula is given by

$$|\epsilon_s| = |\frac{nh^5}{180}f^{iv}(\xi)|, (x_0 < \xi < x_n)$$

where it is assumed that $f^{iv}(\xi)$ is the largest value of the fourth order derivatives.

Example Use Simpson's one-third rule and Trapezoidal rule is found to be equal evaluate $\int_0^6 \frac{dx}{(1+x)^2}$ taking six sub-intervals correct to three decimal places.

solution here, $f(x) = \frac{1}{(1+x)^2}, a = 0, b = 6$, $n = 6$, so $h = \frac{6-0}{6} = 1$.

The interval $[0, 6]$ is divided into six sub-intervals, each of equal length 1, by the points $0, 1, 2, 3, 4, 5, 6$.

The values of $f(x) = \frac{1}{(1+x)^2}$ are calculated for $x = 1, 2, 3, 4, 5, 6$ and tabulated in the next page.

x	0	1	2	3	4	5	6
y	1	0.25	0.1111	0.0625	0.04	0.0278	0.0204

By Simpson's one third rule,

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$$\begin{aligned}\int_0^6 \frac{dx}{(1+x)^2} &= \frac{1}{3}h[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)] \\ &= \frac{1}{3}[1.0204+4\times 0.3403+2\times 0.1511] \\ &= \frac{1}{3}\times 2.6838 = 0.895.\end{aligned}$$

By Trapizoidal Rule,

$$\begin{aligned}\int_0^6 \frac{dx}{(1+x)^2} &= \frac{1}{2}h[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)] \\ &= \frac{1}{2}\times[1.0204+2\times(0.25+0.1111+0.0625+0.04+0.0278)] \\ &= \frac{1}{2}[1.0204+2\times 0.4914] = 1.002\end{aligned}$$

Exercise

1. Evaluate $\int_1^2(x+\frac{1}{x})dx$ upto two significant figures by Simpson's one -third method taking 4 sub intervals.

2. Calculate $\int_0^6 x(1+x)dx$ by Trapezoidal rule, dividing the interval in 6 equal parts.

3. Apply Trapezoidal rule and Simpson's one-third formula to compute $\int_0^{\frac{\pi}{2}} \sin(x)dx$ using the table

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$\sin x$	0	0.25882	0.5	0.70711	0.86603	0.96593	1

4. Use the following data to evaluate $\int_0^1 \cos x dx$, correct to three significant figures

x	0	0.2	0.4	0.6	0.8	1
$\cos x$	1	0.9798	0.9199	0.8228	0.6924	0.5340

5. A curve passes through the points $(1,5),(2,7),(3,9),(4,11),(5,10)$. find the area of the region bounded by the curve, the x axis, the lines $x=1, x=5$.