Lecture : 03

B. Sc. (Hon.) Part-I

Paper - II

Curvature

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I. CURVATURE AT THE ORIGIN

(i). Method of substitution. Radius of curvature at the origin can be found by substituting $x = 0, y = 0$ in the value of ρ obtained or by directly substituting the values of $(y_1)_0$ and $(y_2)_0$ in the formula.

(ii). Method of Expansion In some cases the above method fails, or becomes laborious. In such cases, the values of $(y_1)_0$ and $(y_1)_0$ can be easily obtained in the following way by assuming the equation of the curve to be $y = f(x)$ and writing for y in the equation its expansion by Maclaurin's theorem

$$xf'(0) + \frac{x^2}{2!}f''(0) + ...[f(0) \ being \ zero \ , since \ thecurvepassestheorigin.]$$

Newton's Formula If the curve passes through the origin and the axis of x is the tangent at the origin, we have

$$x = 0, y = 0, (y_1)_0, i.e., p = 0.$$

by Maclaurin's theorem,

$$y = qx^2/2! + ...$$

Dividing by $x^2/2!$ and taking limits as $x \to 0$, we get

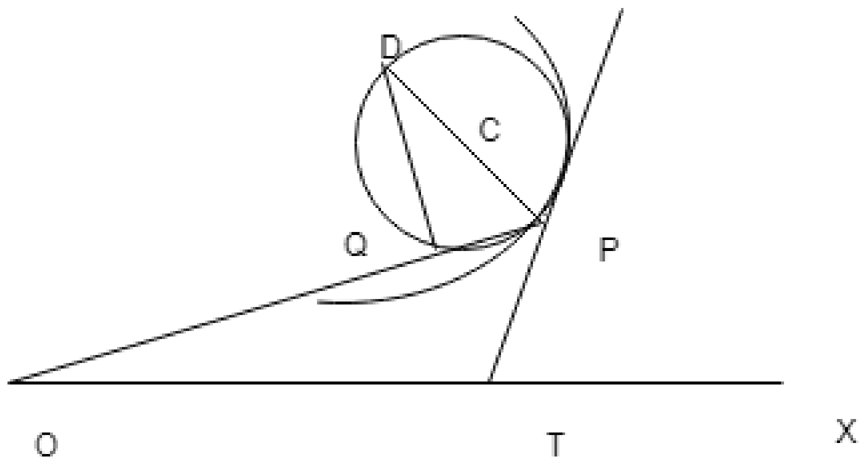
$$\lim(2y/x^2) = q$$

It should be noted here that as $x \to 0$, also $y \to 0$. But from formula at the origin $\rho = \frac{(1+p^2)^{\frac{3}{2}}}{q} = \frac{1}{q}$.

$\rho = \lim_{x,y \to 0} \frac{x^2}{2y}$.

Similarly, if a curve passes through the origin and the axis of y is the tangent there, we have at the origin

$$\rho = \lim_{x,y \to 0} \frac{y^2}{2x}$$



II. ANALYTICALLY:

The equation of the circle passing through the origin and having the x-axis as the tangent at the origin is

$$x^2 + y^2 - 2fy = 0 \tag{1}$$

If r be the radius of the circle, then $r = f$. Since the curve passes through the point (x, y) on the curve, $x^2 + y^2 - 2fy = 0$, whence $f = (x^2 + y^2)/(2f)$. Hence $\rho = \lim r = \lim f = \lim \frac{x^2+y^2}{2y} = \lim \frac{x^2}{2y}$

III. CHORD OF CURVATURE

Chord of curvature through the origin (pole) Let PQ be a chord passing through the origin O of the circle of curvature at P on the given curve, and let O be centre of curvature and PT be the tangent at P. Then $\angle PQD = art\angle, beinginasemicircle$. Then $\angle OPT = \phi and \angle PTX = \psi$. From ΔPQD, chord PQ=PDcos $\angle DPQ$
$= 2\rho \cos\left(\frac{\pi}{2} - \phi\right)$
$= 2\rho \sin\phi$

$= 2.r.\frac{dr}{dp}.\frac{p}{r}$

$= 2p\frac{dr}{dp}$