Lecture : 10

B. Sc. (Hon.) Part-III

Paper - V

# Physics Course: Classical Mechanics

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## I. HAMILTON'S PRINCIPLE

The Hamilton principle requires that a system moves in such a way that the time integral over the Lagrangian $L$ takes an extreme value. This means

$$S = \int_{t_1}^{t_2} L dt$$

shall have an extremum. In other words,

$$\delta S = \int_{t_i}^{t_f} \delta L dt = 0.$$

The path equation of the system can be determined by applying this principle.

## II. PRINCIPLE OF LEAST ACTION

The actual path taken by the system is an extremum of $S$.

## III. EULER-LAGRANGE'S EQUATION

Consider varying a given path slightly, so

$$x^a(t) \rightarrow x^a(t) + \delta x^a(t) \tag{1}$$

where we fix the end points of the path by demanding $x^a(t_i) = x^a(t_f) = 0$. Then the change in the action is

$$\begin{aligned} \delta S &= \int_{t_i}^{t_f} \delta L dt \\ &= \int_{t_i}^{t_f} \left( \frac{\delta L}{\delta x^a} \delta x^a + \frac{\delta L}{\delta \dot{x}^a} \delta \dot{x}^a \right) dt \end{aligned} \tag{2}$$

At this point we integrate the second term by parts to get

$$\delta S = \int_{t_i}^{t_f} \left( \frac{\partial L}{\partial x^a} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}^a} \right) \right) \delta x^a dt + \left[ \frac{\partial L}{\partial \dot{x}^a} \delta x^a \right]_{t_i}^{t_f} \tag{3}$$

But the final term vanishes since we have fixed the end points of the path so $\delta x^a(t_i) = \delta x^a(t_f) = 0$. The requirement that the action is an extremum says that $\delta S = 0$ for all changes in the path $\delta x^a(t)$. We see that this holds if and only if

$$\frac{\partial L}{\partial x^a} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^a}\right) = 0$$

These are known as Lagrange's equations (or sometimes as the Euler-Lagrange equations).