Lecture : 15

B. Sc. (Hon.) Part-I

Paper - I

Physics Course: Waves and Vibration

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I. RESTORING FORCE

A body of mass m lies on a frictionless horizontal surface. It is connected to one end of a spring of negligible mass ,whose other end is fixed to a rigid wall.

If the body is given a displacement along the x-axis and released, it will oscillate back and forth in a straight line along x-axis about the equilibrium position. Suppose at any instant of time the displacement of the body is x from the equilibrium position. There is a force tending to restore the body to its equilibrium position. This force is called the restoring force or return force.

II. FREE OSCILLATION (SIMPLE HARMONIC MOTION)

If the restoring force of a vibrating or oscillatory system is proportional to the displacement of the body from its equilibrium position and is directed opposite to the direction of displacement, the motion of the system is simple harmonic. Therefore,

$$m\ddot{x} = -kx$$

where k is called the spring constant or stiffness factor. The solution of this equation is given by

$$x(t) = A\cos\omega t \tag{1}$$

where $\omega = \sqrt{k/m}$ and A is called the amplitude of the motion. The general solution is given by

$$x(t) = C\cos\omega t + D\sin\omega t \tag{2}$$

where C and D are determined from the initial conditions.

If T is the time for one complete oscillation, then

$$\begin{aligned} x(t+T) &= xt \\ A\cos\omega(t+T) &= A\cos\omega t \\ \omega T &= 2\pi \\ T = \frac{2\pi}{\omega} &= 2\pi\sqrt{\frac{m}{k}} \end{aligned} \tag{3}$$

III. DAMPED HARMONIC MOTION

Various frictional forces may act on a harmonic oscillator, tending to reduce its successive amplitudes. Such a motion is called damped harmonic motion.

Suppose a particle of mass m is subject to a restoring force proportional to the distance from a fixed point on the x-axis and a damping force proportional to the velocity. Then the equation of motion becomes

$$m\ddot{x} = -kx - \beta\dot{x} \tag{4}$$

x being the displacement of the particle from the fixed point at any instant t. The damping force is $-\beta\dot{x}$ where β is the damping coefficient. This equation can be written as

$$\ddot{x} + 2b\dot{x} + \omega^2 x = 0 \tag{5}$$

where $2b = \beta/m$ and $\omega = \sqrt{k/m}$ is the natural frequency of the oscillator. The relaxation time τ is defined by

$$\tau = \frac{1}{2b} = \frac{m}{\beta} \tag{6}$$