Lecture : 07

B. Sc. (Hon.) Part-III

Paper - V

Physics Course: Methods of Mathematical Physics

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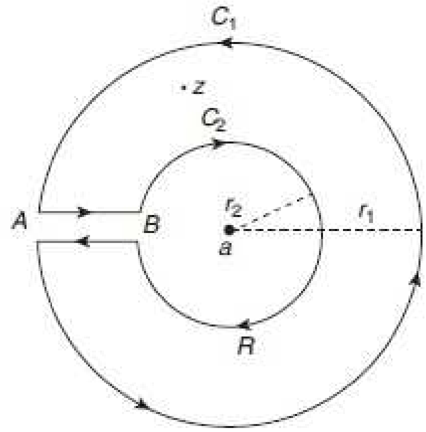

I. LAURENT'S THEOREM

If $f(z)$ is analytic on two concentric circles C_1 and C_2 with the centre at $z = a$ and radii $r_1, r_2 (r_2 < r_1)$ and in the ring shaped (annular) region $\mathcal{R}$ between C_1 and C_2 then for all z in $\mathcal{R}$, we have

$$\begin{aligned} f(z) &= a_0 + a_1(z-a) + a_2(z-a)^2 + ... + \frac{b_1}{z-a} + \frac{b_2}{(z-a)^2} + ... \\ &= \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-a)^n} \end{aligned} \quad (1)$$

where

$$\begin{aligned} a_n &= \frac{1}{2\pi i} \int_{C_1} \frac{f(z)}{(z-a)^{n+1}} dz \\ b_n &= \frac{1}{2\pi i} \int_{C_2} \frac{f(z)}{(z-a)^{-n+1}} dz \end{aligned}$$

The part $a_0 + a_1(z-a) + a_2(z-a)^2 + ...$ is called the analytic part of the Laurent series, while the remaining part of the series $\frac{b_1}{z-a} + \frac{b_2}{(z-a)^2} + ..$ is called the principal part. When the principal part is zero, the Laurent series reduces to a Taylor series.

II. CAUCHY'S INTEGRAL THEOREM

Let $f(z)$ be analytic in a region $\mathcal{R}$ and on its boundary C. Then

$$\oint_C f(z)dz = 0.$$

This is a fundamental theorem known as Cauchy's integral theorem. This theorem is valid for both simply- and multiply-connected regions.

III. RESIDUE THEOREM

Let $f(z)$ be single-valued and analytic inside and on a simple closed curve C except at the singularities $a, b, c, ...$ inside C, which have residues given by $a_{-1}, b_{-1}, c_{-1}, ...$, Then, the *residue theorem* states that the integral of $f(z)$ around C is $2\pi i$ times the sum of the residues of $f(z)$ at the singularities enclosed by C, i.e.,

$$\oint_C f(z)dz = 2\pi i(a_{-1} + b_{-1} + c_{-1}).$$