Lecture : 05

Physics Course: Methods of Mathematical Physics

For B. Sc. (Hon.) III

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I. COMPLEX VARIABLES.

A symbol z which can stand for any one of a set of complex numbers is called a complex variable.

$$z = a + ib,$$

where a and b are real numbers.

II. FUNCTIONS OF COMPLEX VARIABLES

If for each value that a complex variable z, there exist one or more values of a complex variable w, then w is a function of z and can be expressed as $w = f(z)$. The variable z is sometimes called an independent variable, while w is called a dependent variable. There are many elementary functions.

A. Polynomial function

Polynomial functions are defined by

$$w = a_0 z^n + a_1 z^{n-1} + ... + a_n = P(z),$$

where $a_0 \neq 0, a_1, ..., a_n$ are complex constants and n is a positive integer called the degree of the polynomial $P(z)$.

B. Rational algebraic function

Rational algebraic functions are defined by

$$w = \frac{P(z)}{Q(z)},$$

where $P(z)$ and $Q(z)$ are polynomials.

C. Exponential function

Exponential functions are defined by

$$w = e^z = e^{x+iy} = e^x(\cos y + i \sin y).$$

D. Trigonometric function

Trigonometric or circular functions are defined as $\sin z, \cos z, \tan z$, etc.

E. Hyperbolic function

Hyperbolic functions are defined as $\sinh z, \cosh z, \tanh z$, etc.

F. Logarithmic function

TChe natural logarithmic function is the inverse of the exponential function and can be defined by

$$w = \ln z.$$

G. Algebraic and transcendental functions functions

If w is a solution of the polynomial equation

$$P_0(z)w^n + P_1(z)w^{n-1} + ... + P_{n-1}(z)w + P_n(z) = 0 \tag{1}$$

where $P_0 \neq 0, P_1(z), ..., P_n(z)$ are polynomials in z and n is a positive integer, then $w = f(z)$ is called an algebraic function of z.

Any function that cannot be expressed as a solution of above polynomial equation is called a transcendental function. The logarithmic, trigonometric, and hyperbolic functions and their corresponding inverses are examples of transcendental functions.

III. ANALYTIC FUNCTIONS

If the derivative $f'(z)$ exists at all points z of a region $\mathcal{R}$ of the z plane, then $f(z)$ is said to be analytic function in $\mathcal{R}$.

A function $f(z)$ is said to be analytic at a point z_0 if there exists a neighborhood $|z - z_0| < \delta$ at all points of which $f'(z)$ exists.