Lecture #: 01

Physics Course: Relativity

For B. Sc. (Hon.) and B. Sc. (IT)

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I. COORDINATE SYSTEMS

Purpose of a Coordinate Systems: The purpose of a coordinate system is to uniquely determine the position of an object in space. A good understanding of coordinate systems can be very helpful in solving many problems in physics.

A coordinate system consists of four basic elements:

1. Choice of origin
2. Choice of axes
3. Choice of positive directions for each axis
4. Choice of unit vectors for each axis

The three most common coordinate systems are: Cartesian, Cylindrical and Spherical coordinates.

A. Cartesian Coordinates

1. **Choice of origin**: Choose an origin O. If you are given an object, then your choice of origin may coincide with a special point in the body. For example, you may choose the mid-point of a straight piece of wire.

2. **Choice of axis**: Now we choose a set of axes. The simplest set of axes, known as the Cartesian axes, are x-axis, y-axis, and the z-axis. Then each point in space P can be assigned (x, y, z), the Cartesian coordinates of the point. The ranges of these values are: $-\infty < x < +\infty$, $-\infty < y < +\infty$, $-\infty < z < +\infty$.

3. **Choice of Positive Direction**: Our third choice is an assignment of positive direction for each coordinate axis. Conventionally, Cartesian coordinates are drawn to the plane of the paper. The horizontal direction from left to right is taken as the positive x-axis, and the vertical direction from bottom to top is taken as the positive y-axis. In physics problems we are free to choose our axes and positive directions any way that we decide best fits a given problem.

4. **Choice of Unit Vectors**: We now associate to each point in space, a set of three unit directions vectors $(\hat{\imath}, \hat{\jmath}, \hat{k})$. A unit vector has magnitude one.

In Cartesian coordinates an infinitesimal displacement vector ds between two points $P1$ and $P2$ can be decomposed into

$$d\vec{s} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}. \tag{1}$$

An infinitesimal length in such coordinate system is given by

$$dL = |d\vec{s}| = \sqrt{dx^2 + dy^2 + dz^2}. \tag{2}$$

and an infinitesimal volume by

$$dv = dxdydz. \tag{3}$$

B. Cylindrical Coordinates

We first choose an origin O and an axis we call the z-axis with unit vector $\hat{z}$ pointing in the increasing z-direction. Its value ranges from $-\infty \leq \rho < +\infty$. The coordinate ρ measures the distance from the z-axis to the point P. Its value ranges from $0 \leq \rho < +\infty$. We define ϕ as the angle in the counterclockwise direction from x-axis to the line joining from the origin to the projection of point P on xy-plane. ϕ can take value from $0 \leq \phi < 2\pi$.

The coordinates (ρ, ϕ) in the plane z=constant are called *plane polar coordinates*. Our complete coordinate system is shown in Figure. This coordinate system (ρ, ϕ, z) is called a *cylindrical coordinate system*.

If you are given polar coordinates (ρ, ϕ) of a point in the plane, the Cartesian coordinates (x, y) can be determined from the coordinate transformations:

$$x = \rho \cos \phi, \quad y = \rho \sin \phi. \tag{4}$$

Conversely, if you are given the Cartesian coordinates, the polar coordinates may be represented as

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}(y/x) \tag{5}$$

In cylindrical coordinates an infinitesimal displacement vector ds between two points $P1$ and $P2$ can be decomposed into

$$d\vec{s} = d\rho\hat{\varrho} + \rho d\phi\hat{\varphi} + dz\hat{k}. \tag{6}$$

An infinitesimal length in such coordinate system is given by

$$dL = |d\vec{s}| = \sqrt{d\rho^2 + \rho^2 d\phi^2 + dz^2}. \tag{7}$$

and an infinitesimal volume by

$$dv = \rho d\rho d\phi dz. \tag{8}$$

C. Spherical Coordinates

We first choose an origin O. Then we choose a coordinate, r, that measures the radial distance from the origin to the point P. The coordinate r ranges in value from $0 \leq r < +\infty$. The set of points that have constant value for are spheres.

Any point on the sphere can be defined by two angles (θ, ϕ) and r. The angle θ is defined to be the angle between the positive z-axis and the line from the origin to the point P. it ranges $0 \leq \theta \leq \pi$. The angle ϕ is defined in a similar fashion to polar coordinates.

If you are given spherical coordinates (r, θ, ϕ) of a point in the plane, the Cartesian coordinates (x, y, z) can be determined from the coordinate transformations

$$\begin{aligned} x &= r \sin\theta \cos\phi, \\ y &= r \sin\theta \sin\phi, \\ z &= r \cos\theta. \end{aligned} \tag{9}$$

In spherical coordinates an infinitesimal displacement vector ds between two points $P1$ and $P2$ can be decomposed into

$$d\vec{s} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}. \tag{10}$$

An infinitesimal length in such coordinate system is given by

$$dL = |d\vec{s}| = \sqrt{dr^2 + r^2 d\theta^2 d\phi^2 + r^2 \sin^2\theta d\phi^2}. \tag{11}$$

and an infinitesimal volume by

$$dv = r^2 \sin\theta dr d\theta d\phi. \tag{12}$$

II. FRAME OF REFERENCE

Frame of reference is a coordinate system in which you measure the position of the particle. For example, when you perform an experiment in a laboratory, you select a coordinate system, or frame of reference, that is at rest with respect to the laboratory. However, suppose an observer in a passing car moving at a constant velocity with respect to the lab were to observe your experiment.

III. INERTIAL FRAME OF REFERENCE

According to the principle of Galilean relativity, the laws of mechanics must be the same in all inertial frames of reference. Inertial frames of reference are those reference frames in which Newtons laws are valid. In these frames, objects move in straight lines at constant speed unless acted on by a non-zero net force thus the name inertial frame, because objects observed from these frames obey Newtons first law, the law of inertia, i.e. an object moves with a velocity that is constant in magnitude and direction, unless acted on by a nonzero net force.