Lec 01

GROUP THEORY

For B.Sc, B.C.A, B.Sc(I.T)

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1. GROUP STRUCTURE

An algebraic structure is a set of elements (the carrier of the structure) with an operation (equally denoted application) that matches any two members of the set uniquely onto a third member. The specificity of an algebraic structure is given by the axioms that it satisfies. One of the most basic algebraic structures is the group.

(1.1) Definition.

Definition 1.1: *If G is a nonempty set, a binary operation * on G is a function* $* : G \times G \rightarrow G$.

Definition 1.2: *A binary operation * on set G is associative if*

$(a * b) * c = a * (b * c)$, *for all* $a; b; c \in G$.

Definition 1.3: *A group (G; *) is a set G with a special element e on which an associative binary operation * is defined that satisfies:*

1. $e * a = a$ *for all* $a \in G$;

2. *for every* $a \in G$, *there is an element* $b \in G$ *such that* $b * a = e$.

Definition of Group

*A **group** is a couple (G, *) where:*
1) G is a set
*2) * is an application,* $* : G \times G \rightarrow G$, *[i.e. (G, *) is closure property]*
3) $\forall a, b, c \in G$, *the relation* $a*(b*c)=(a*b)*c$ *is fulfilled [Associative Property]*
4) $\exists e \in G$ *such that* $\forall a \in G$, *the relation* $(e*a) = (a*e) = a$ *is fulfilled [Identity property, where e is called identity element]*
5) $\forall a \in G$, $\exists b \in G$ *such that* $a*b=b*a=e$, *where b is called inverse of a or vice versa.*

1.2 Some properties are unique.

Lemma 1.2.1. *If (G, *) is a group and* $a \in G$, *then* $a*a = a$ *implies* $a = e$.

Proof. Suppose a 2 G satisfies a*a = a and let b ϵ G be such that b*a = e.
Then b * (a* a) = b * a and thus

a = e * a = (b * a) * a = b * (a * a) = b * a = e.

Lemma 1.2.2. *In a group (G,*)*
*(i) if b * a = e, then a * b = e and*

*(ii) a * e = a for all a ϵ G*

Furthermore, there is only one element e ϵ G satisfying (ii) and for all

*a ϵ G, there is only one b ϵ G satisfying b * a = e.*

Proof. Suppose b * a = e, then
(a * b) * (a * b) = a * (b * a) * b = a * e * b = a * b:
Therefore by Lemma 1.2.1 a * b = e.

Suppose a ϵ G and let b ϵ G be such that b * a = e. Then by (i)
a * e = a * (b * a) = (a * b) * a = e * a = a
Now we show uniqueness. Suppose that a * e = a and a * f = a for all

a ϵ G. Then
(e * f) * (e * f) = e * (f * e) * f = e * f * e = e * f
Therefore by Lemma 1.2.1 e * f = e. Consequently
f * f = (f * e) * (f * e) = f * (e * f) * e = f * e * e = f * e = f
and therefore by Lemma 1.2.1 f = e. Finally suppose b1 * a = e and
b2 * a = e. Then by (i) and (ii)
b1 = b1 * e = b1 * (a * b2) = (b1 * a) * b2 = e * b2 = b.

1.2. SOME PROPERTIES ARE UNIQUE.

Definition 1.4: *Let (G; *) be a group. The unique element e ϵ G satisfying e * a = a for all a ϵ G is called the identity for the group (G; *). If a ϵ G, the unique element b * G such that b * a = e is called the inverse of a and we denote it by* $b = a^{-1}$.

Definition 1.5: *A Latin square of side n is an n by n array in which each cell contains a single element form an n-element set S = {s1; s2;…; sn}, such that each element occurs in each row exactly once. It is in standard form with respect to the sequence s1; s2; … ; sn if the elements in the first row and first column are occur in the order of this sequence.*