Lecture : 01

B. Sc. (Hon.) Part-I

Paper - II

Partial Differential Equation

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I. FORMATION OF PDE

A. Formation of PDE by eliminating arbitrary constant

1. Eliminate the arbitrary constant a, b from $x^2 + y^2 + (z - b)^2 = a^2$ Differentiating partally w.r.t x and y respectively we get,

$$2x + 2(z - b)p = 0 \quad (1)$$

$$2y + 2(z - b)q = 0 \quad (2)$$

Multiply the first equation by q and second equation by p and substituting we get

$$qx - py = 0 \quad (3)$$

This is the required PDE.

2. Eliminating a,b,c from the relation $z = ax + by + cxy$ *and from the PDE.* Differentiating partally w.r.t x and y respectively we get,

$$p = a + cy \quad (4)$$

$$q = b + cx \quad (5)$$

Thses equation together with the given equation are not sufficient for the elimination of a, b, c. We therefore consider second order partial derivatives. Differentiating partially (1) w.r.t y and (2) we get,

$$s = c \quad (6)$$

$$p = a + sy \quad (7)$$

$$q = b + sy \quad (8)$$

Substituting these value of a,b from the above relation we get

$$z = px + qy - sxy \quad (9)$$

Differentiating partially (1) and (2) w.r.t x and y respectively we get ,

$$r = 0, \quad t = 0 \tag{10}$$

3. Find all the possible of least order corresponding to each of the following relations. The letters a,b,c denote arbitrary constants to be eliminated.

i. $(x-a)^2 + (y-b)^2 + z^2 = 1$

$$(x-a)^2 + (y-b)^2 + z^2 = 1 \tag{11}$$

Differentiating equation (1) w.r.t x we get

$$2(x-a) + 2zp = 0 \tag{12}$$

$$2(t-b) + 2zq = 0 \tag{13}$$

From the equation (2) we get $a = x + zp$ and $b = y + zq$. Substituting a and b from (2) and (3) in (1) we get

$$z^2(p^2 + q^2 + 1) = 1 \tag{14}$$

iii. $ax + by + cz = 1$

$$ax + by + cz = 1 \tag{15}$$

Differentiating partially w.r.t x and y we get,

$$a + cp = 0 \tag{16}$$

$$b + cq = 0 \tag{17}$$

These two equations is not sufficient condition for eliminating of arbitrary constant from (2) and (3). then we use the higher order partially differentiation we get (2) w.r.t x and (3) w.r.t y we get

$$cr = 0 \implies r = 0 \tag{18}$$

iv. z=$ax^2 + by^2 + ab$*to* $z = ax^2 + by^2 + ab$(19)Differentiating w.r.t x and y respectively we get

$$p = 2ax \tag{20}$$

$$q = 2by \tag{21}$$

Substituting a, b from (2) and (3) in equation (1) we get,

$$z = \frac{px}{2} + \frac{qy}{2} + \frac{pq}{4xy} \tag{22}$$

$$\implies 4xyz = 2px + 2qy + pq \tag{23}$$

v. $z = ax^2 + bxy + cy^2 + d$

$$z = ax^2 + bxy + cy^2 + d \tag{24}$$

Differentiating partially w.r.t x and y respectively we get

$$p = 2ax + by \tag{25}$$

$$q = bx + 2cy \tag{26}$$

These two equations are not sufficient condition for eliminating of arbitrary constant. Then we use higher order partial differential equation. By differentiating (2) w.r.t y and (3) w.r.t x we get

$$s = b \tag{27}$$

Again differentiating (2) partially w.r.t x and y we get,

$$r = 2a, t = 2c. \tag{28}$$

Substituting a,b,c, in equation (1) we get

$$z = \frac{rx^2}{2} + sxy + \frac{ty^2}{2} \tag{29}$$