Lecture : 02

B. Sc. (Hon.) Part-I

Paper - II

Tangent & Normal

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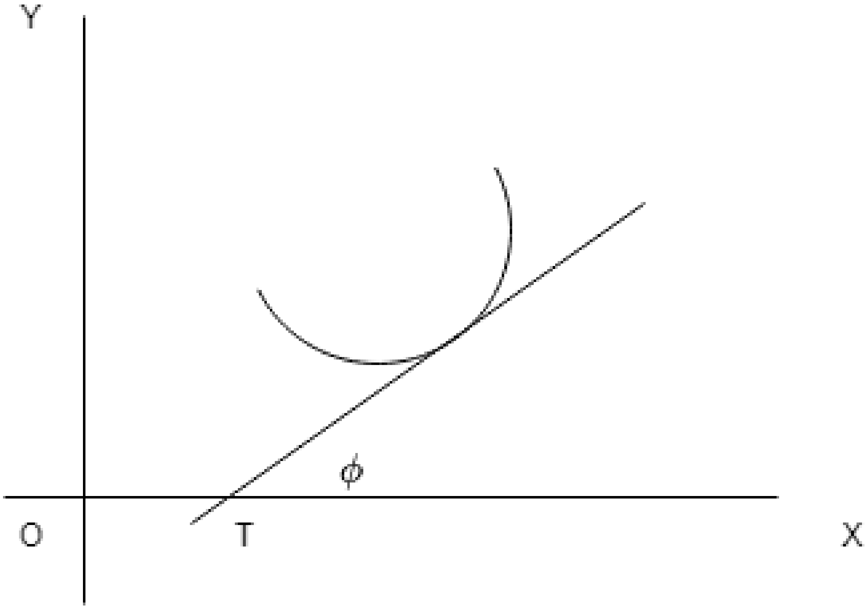
I. GEOMETRICAL MEANING OF $\frac{dy}{dx}$



The equation of the tangent can be written as

$$Y = \frac{dy}{dx}.X + (y - x\frac{dy}{dx})$$

which being of the form $y = mx + c$, the standard equation of a straight line, we conclude that $\frac{dy}{dx}$ is the m of the tangent at (x,y).

If ϕ be the angle which the positive direction of the tangent at P makes with the positive direction of the x -axis, then $tan\phi = m = \frac{dy}{dx}$.

Hence, the derivative $\frac{dy}{dx}$ at (x,y)is equal to the trigonometrical tangent of the couple which the tangent to the curve at (x,y) makes with the positive direction of the x-axis.

Note 1. It is customary to denote the angle which the tangent at any point on a curve makes with the x-axis by ϕ.

Note 2. The tangent direction of the tangent is the direction of the arc length s increasing.

Note 3. $tan\phi \;\; i.e. \frac{dy}{dx}$ is also called the gradient of the curve at the point $P(x, y)$.

Note 4. The tangent at (x, y) is parallel to the x axis if $\phi = 0$, i.e., if $tan\phi = 0, \;\; i.e. \frac{dy}{dx} = 0$.

II. TANGENT AT THE ORIGIN:

If a curve passing through the origin be given by a rational integral algebraic equation, the equation of the tangent at the origin i obtained by equation to zero the terms of the lowest degree in the equation.

Let the equation of a curve of the nth degree passing through the origin be

$$a_1x + b_1y + a_2x^2 + b_2xy + c_2y^2 + ... + a_nx^n + ... + k_ny^n = 0$$

Let $P(x, y)$ be a point on the curve near the origin O.

The equation of the scant $\bar{O}P$ is $Y = \frac{y}{x}X$.

the equation of the tangent at O is

$$Y = \lim_{x \to 0_{y \to 0}} \frac{y}{x} X = mX$$

Thus, the 'm' of the tangent at the origin is $\lim_{x \to 0} \frac{y}{x}$.

CASE I. Let us support that m is finite, i.e., the y axis is not tangent at the origin.

i Let us suppose $b_1 \neq 0$.

dividing (1) by x, we get

$$a_1 + b_1\frac{y}{x} + a_2x + b_2y + c_2y\frac{y}{x} + ... = 0$$

Now, let $x \to 0, \; y \to 0$, then $\lim(\frac{y}{x}) = m$.

$a_1 + b_1 m = 0$, the other terms vanishing.

$m = -\frac{a_1}{b_1}$.

From the above equations of the tangent at the origin is $a_1 X + b_1 Y = 0$,

or, taking x and y as current co-ordinates,

$a_1 x + b_1 y = 0$

ii. If $b_1 = 0$, then it follows that $a_1 = 0$; now in this case, we can written as

$$a_2 x^2 + b_2 xy + c_2 y^{+} a_3 x^{+} ... = 0$$

Diving by x $a_2 + b_2 \frac{y}{x} + c_2 (\frac{y}{x})^2 + a_3 x + .. = 0$.

When $x \to 0$, $y \to 0$, we have

$a_2 + b_2 m + c_2 m^2 = 0$, the other terms vanishing.

It is clear that there are two values of m and hence, there are two tangents at the origin and their equation which is obtained by elimination m we get $a_2 X^2 + b_2 XY + c_2 Y^2 = 0$ or, taking x and y as current co-ordinates,

$a_2 x^2 + b_2 xy + c_2 y^2 = 0$.

If $a_1 = b_1 = a_2 = b_2 = c_2 = 0$, it can be shown similarly that the rule holds good then also and so on.

Case II. When the tangent at the origin is the y-axis, then $\lim \frac{x}{y}$, as x and y both tends to 0, being the tangent of the inclination of the tangent at the origin to the y axis, is zero.

Hence dividing throughout the equation of the curve by y, and assuming $a_1 \neq 0$, and making x and y both approaches to zero, we find $b_1 = 0$. Hence the equation become $a_1 x + a_2 x^2 + b_2 xy + c_2 y^2 + ... = 0$.

We see that the theorem is still true in this case.