MATRICES (PAPER 1 GROUP B)- FIRST YEAR

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1 Some typical type of Matrices

(1. Idempotent matrix:

A square matrix A is said to be idempotent matrix if $A^2 = A$.

e.g. matrix $A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$ **is idempotent, for**

$A^2 = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} \times \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix} = A$. **2. Nilpotent Matrix:**

A square matrix A is said to be nilpotent matrix of index k, if k be the least positive integer for which $A^k = O$, null matrix.

e.g. $A = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}$ **is a nilpotent matrix of index 2, for**

$A^2 = \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O$

3. Involutary matrix:

A square matrix A is said to be involutary matrix if $A^2 = I$.

e.g. the matrix $A = \begin{pmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{pmatrix}$ **is a involutary matrix for**

$A^2 = \begin{pmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{pmatrix} \times \begin{pmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

2 Transpose of a Matrix

Let A be a matrix of size $m \times n$. Then the matrix A^T obtained by interchanging the rows and columns of A is called transpose of A. It

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is also denoted as A'. Thus $A = (a_{ij})_{m,n}$ then its transpose $A^T = (a'_{ij})_{mn}$ where $a'_{ij} = a_{ji}$.

If $A = \begin{pmatrix} 2 & 5 & 6 \\ -1 & 2 & 0 \end{pmatrix}$ then its transpose $A^T = \begin{pmatrix} 2 & -1 \\ 5 & 2 \\ 6 & 0 \end{pmatrix}$.

3 Properties of Transpose Matrix:

If A and B are two matrices, then

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$
3. $(A - B)^T = A^T - B^T$
4. $(kA)^T = kA^T$, **k is a number.**
5. $(AB)^T = B^T A^T$

4 Symmetric and Skew-symmetric matrices:

i. A square matrix $A = (a_{ij})_{n\times m}$ is called symmetric matrix if $A^T = A$.

e.g. $A = \begin{pmatrix} 3 & 5 & 9 \\ 5 & 2 & 0 \\ 9 & 0 & 5 \end{pmatrix}$ is a symmetric matrix because here $A^T = A$.

ii. A square matrix $A = (a_{ij})_{n\times m}$ is called symmetric matrix if $A^T = -A$.

e.g. $A = \begin{pmatrix} 0 & 6 & -2 \\ -6 & 0 & -3 \\ 2 & 3 & 0 \end{pmatrix}$ is a symmetric matrix because here $A^T = -A$.

Note: All the diagonal elements of a skew symmetric matrix must be 0, because here $a_{ij} = -a_{ji}$ i.e. $2a_{ii} = 0 \Rightarrow a_{ii} = 0$.

Theorem 1. *Any square matrix can be expressed as a sum of a symmetric and skew symmetric matrix.*

Proof. Let A be a square matrix of size $m \times n$. We can write

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) \qquad (1)$$

Put $P = \frac{1}{2}(A + A^T)$ and $Q = \frac{1}{2}(A - A^T)$.

Then

$$\begin{aligned} P^T &= (\frac{1}{2}(A + A^T))^T \\ &= \frac{1}{2}((A + A^T))^T = \frac{1}{2}(A^T + (A^T)^T) \\ &= \frac{1}{2}(A^T + A) = P \end{aligned}$$

Therefore P is a symmetric matrix.

Now for Q,

$$\begin{aligned} Q^T &= (\frac{1}{2}(A - A^T))^T \\ &= \frac{1}{2}((A - A^T))^T = \frac{1}{2}(A^T - (A^T)^T) \\ &= \frac{1}{2}(A^T - A) = -Q \end{aligned}$$

Therefore Q is a skew-symmetric matrix. Hence the theorem is proved. $\square$